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Seven–Superform Gauge Fields in N=1, D=10 Supergravity and Duality*

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Abstract

We present a formulation of $N = 1, D = 10$ Supergravity–Super–Maxwell theory in superspace in which the graviphoton can be described by a 2–form B_2 or a 6–form B_6, the photon by a 1–form A_1 or a 7–form A_7 and the dilaton by a scalar φ or an 8–form φ_8, the supercurvatures of these fields being related by duality. Duality interchanges Bianchi identities and equations of motion for each of the three couples of fields. This construction envisages the reformulation of $D = 10$ Supergravity, involving 7–forms as gauge fields, conjectured by Schwarz and Sen, which, upon toroidal compactification to four dimensions, gives the manifestly $SL(2, R)_S$ invariant form of the heterotic string effective action.

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Recently Schwarz and Sen [1] got a manifestly general coordinate invariant four dimensional heterotic string effective action which is also manifestly invariant under the $SL(2,R)_S$ duality. This action follows quite naturally from dimensional reduction of the dual version of $D=10$ Supergravity. However, manifest $SL(2,R)_S$ invariance could not be achieved when trying to include the Super–Yang–Mills (SYM) sector in the ten dimensional model. For that, it would be necessary to replace half of the four dimensional scalar and vector fields arising from the abelian, Cartan subalgebra valued, ten dimensional gauge fields with their duals. Schwarz and Sen suggested that these dual fields should arise, via dimensional reduction, from 7–forms in $D=10$. This led them to conjecture that a new formulation of $N=1, D=10$ Supergravity–Super–Maxwell (SUGRA–MAX) should exist where half of the abelian gauge fields are replaced by abelian 7–forms.

This letter provides the main ingredients for this new formulation. We shall work in the framework of the geometric superspace approach (see ref. [2] and references therein). In this approach one introduces supervielbeins, Lorentz and gauge superconnections and, in case, p–superforms, imposes suitable constraints on their curvatures and solves the Bianchi identities (B.I.) under these constraints to get the equations of motion and the supersymmetry transformation laws for the physical fields.

A recent discussion of $N=1, D=10$ Supergravity models in this framework has been made in [3]. In this letter we shall follow the notations of that paper. In particular, superspace is described locally by the supercoordinates $Z^M=(X^m,\theta^\mu)$ where X^m $(m=0,\cdots,9)$ are space–time coordinates and θ^μ $(\mu=1,\cdots,16)$ are Grassmann variables. Latin and Greek letters denote respectively vector–like and spinor–like indices and Capital letters both kind of indices. Letters from the beginning of the alphabet are kept for the (co) tangent superspace. Given a local frame specified by the one superforms $E^A(Z)$ and a p–superform Ψ_p

$$\Psi_p=\frac{1}{p!}E^{A_1}\cdots E^{A_p}\Psi_{A_p\cdots A_1}$$

it will be useful to call $(q,p-q)$ sector of Ψ_p, denoted by $\Psi_{q,p-q}$, the component of Ψ_p proportional to q vector–like supervielbeins E^a and $p-q$ spinor–like supervielbeins E^α. Moreover, d is the superspace differential and D is the Lorentz–covariant one.

In the standard superspace formulation of $N=1$, $D=10$ SUGRA–SYM one introduces the supervielbeins $E^A(Z)$, the Lorentz valued superconnection $\Omega_A{}^B(Z)$ the Lie algebra valued gauge superconnection $A_1(Z)$ and the 2–superform $B_2(Z)$.

Their curvatures are respectively the torsion $T^A = DE^A$, the Lorentz curvature $R_A{}^B$, the gauge curvature F_2 and the B_2–curvature $H_3 = dB_2 + \Omega_3$. The 3–superform Ω_3 depends on the model. For instance it vanishes in pure supergravity and it is proportional to the Chern–Simons 3–superform associated to A_1 in the minimally coupled SUGRA–SYM [4]. The theory is set on shell by imposing suitable constraints on the lower dimension sectors of torsion, gauge curvature and B_2–curvature [5].

In the dual formulation [6] B_2 is replaced by a 6–superform B_6. However in this case the theory can not be put on-shell by imposing suitable constraints on the B_6–curvature, $H_7 = dB_6$, in addition to the standard torsion and gauge curvature constraints [7]. For that one has to fix a superfield, which belongs to the 120 irreducible representation (irrep.) of $SO(1,9)$, which can not be determined by solving the B.I., see [8,9]. For a recent discussion of the dual formulation see [9].

A more symmetric situation arises in the approach of ref. [3]. Here the superforms B_2 and B_6 are not even introduced from the beginning and the theory is set on shell by adding to the usual torsion and gauge curvature constraints a further constraint on the $(0,2)$ sector of the Lorentz curvature. It has been shown in [3] that under these constraints one can define a 3–superform H_3 and a 7–superform H_7 that satisfy suitable identities such that H_3 (H_7) can be considered locally as the curvature of a 2–superform B_2 (6–superform B_6). (The asymmetry between the two possibilities, however, remains since H_7 is always closed whereas H_3 is closed only in pure supergravity).

In this letter we will show that a situation similar to the one described in [3] for the B–sector arises in the gauge sector and in the dilaton sector. We shall discuss the minimally coupled SUGRA–MAX theory with abelian gauge group $U(1)^{2n}$ (in the case of the heterotic string effective action one has to set $n = 8$). The gauge curvatures are

$$\vec{F}_2 = d\vec{A}_1 \tag{1}$$

with the B.I.

$$d\vec{F}_2 = 0. \tag{2}$$

Here $\vec{A}_1 \equiv (A_1^{(1)}, \cdots, A_1^{(2n)})$ denotes the set of Maxwell one–superforms. In this case the identity for H_3 is

$$dH_3 = \gamma \vec{F}_2 \cdot \vec{F}_2 \tag{3}$$

so that locally

$$H_3 = dB_2 + \gamma \vec{A}_1 \cdot \vec{F}_2, \tag{4}$$

and the identity for H_7 is

$$dH_7 = 0 \tag{5}$$

so that locally

$$H_7 = dB_6. \tag{6}$$

We shall show that under the constraints of ref. [3] it is possible to reconstruct $2n$ eight–superforms $\vec{F}_8$, dual to $\vec{F}_2$, $\vec{F}_8 \equiv (F_8^{(1)}, \cdots, F_8^{(2n)})$, that satisfy the identities

$$d\vec{F}_8 = \vec{F}_2 H_7, \tag{7}$$

so that locally there exist $2n$ 7–superforms $\vec{A}_7$ (Z) such that

$$\vec{F}_8 = d\vec{A}_7 + \vec{F}_2 B_6. \tag{8}$$

In a formulation where $\vec{A}_1$ are the relevant gauge fields the sector (3,0) of eq. (2) describes the B.I. for their curvatures and the sector (9,0) of eq. (7) provides their field equations. However, according to eq. (8), we propose other formulations in which some of the one–superforms $\vec{A}_1$ can be replaced by the corresponding 7–superforms $\vec{A}_7$, in agreement with the conjecture of ref. [1]. In this case (7) becomes a B.I. and (2) has to be read as equation of motion for $\vec{A}_7$.

In addition we shall show that it is possible to reconstruct a 9–superform V_9, dual to the curvature V_1 of the dilaton φ, $V_1 = d\varphi$, that satisfies the identity

$$dV_9 = H_3 H_7 + \gamma \vec{F}_2 \cdot \vec{F}_8, \tag{9}$$

in such a way that the highest sector (10, 0) of eq. (9) is just the field equation for φ (a 0–superform). Again, (9) implies locally the existence of an 8–superform φ_8 such that

$$V_9 = d\varphi_8 - B_2 H_7 + \gamma \vec{A}_1 \cdot \vec{F}_8, \tag{10}$$

or, alternatively, of an 8–superform $\hat{\varphi}_8$ such that

$$V_9 = d\hat{\varphi}_8 + H_3 B_6 + \gamma \vec{F}_2 \cdot \vec{A}_7. \tag{11}$$

If (9) is regarded as the B.I. for $(\varphi_8, \hat{\varphi}_8)$ its equation of motion becomes just

$$dV_1 = 0. \tag{12}$$

To prove these results we shall work for simplicity in the SUGRA–MAX theory with only one gauge multiplet, the generalization to the case with $2n$ gauge multiplets being straightforward. The relevant B.I. to be considered are the torsion B.I.

$$DT^A = E^B R_B{}^A \tag{13}$$

and the gauge B.I.

$$dF_2 = 0. \tag{14}$$

The torsion and curvature constraints, chosen in [3], are

$$T_{\alpha\beta}{}^a = 2(\Gamma^a)_{\alpha\beta}, \qquad T_{a\alpha}{}^b = 0 = T_{\alpha a}{}^b \tag{15}$$

$$F_{\alpha\beta} = 0 \tag{16}$$

$$R_{\alpha\beta ab} = (\Gamma_{[a}\Gamma^{cde}\Gamma_{b]})_{\alpha\beta} J_{cde} * . \tag{17}$$

Notice that, apart from (13)–(17), we do not impose constraints or B.I on any other superform. All our results will be obtained by demanding the closure of the SUSY–algebra on (13)–(17)
We defined $\Gamma^{a_1\cdots a_k} \equiv \Gamma^{[a_1}\Gamma^{a_2}\cdots\Gamma^{a_k]}$ and $(\Gamma^a)_{\alpha\beta}, (\Gamma^a)^{\alpha\beta}$ are Weyl matrices in $D = 10$. The current J^{cde}, a 120 irrep. of $SO(1,9)$, is a local function of the relevant superfields and (13) demands that in the covariant spinorial derivative of $e^{2\varphi} J_{cde}$, the highest irrep., i.e. the 1200, is absent. The explicit expression of J_{cde} depends on the model considered. In the present case it is given by

$$J_{cde} = -\frac{\gamma}{12}(\Gamma_{cde})_{\alpha\beta}\chi^\alpha\chi^\beta \equiv -\frac{\gamma}{12}\chi_{cde} \tag{18}$$

and it can be verified that it satisfies the just mentioned condition. Here χ^α is the gravitino superfield which is present in the (1,1) sector of the gauge curvature

* This constraint differs from the one in [3] by a shift of the Lorentz connection.

$$F_{a\alpha} = 2(\Gamma_a)_{\alpha\beta}\chi^\beta \tag{19}$$

as a consequence of the constraint (16) and the B.I. (14). γ is a coupling constant which vanishes in pure Supergravity. From the B.I. (13) one has also

$$T_{\alpha\beta}{}^\gamma = 2\delta^\gamma_{(\alpha}V_{\beta)} - (\Gamma^a)_{\alpha\beta}(\Gamma_a)^{\gamma\delta}V_\delta \tag{20}$$

$$T_{a\alpha}{}^\beta = \frac{1}{4}(\Gamma^{bc})_\alpha{}^\beta T_{abc} - 2\gamma(\Gamma_a)_{\alpha\gamma}\chi^\gamma\chi^\beta \tag{21}$$

$$R_{a\alpha bc} = 2(\Gamma_a)_{\alpha\beta}T_{bc}{}^\beta + 6\gamma(\Gamma_{[a})_{\alpha\beta}F_{bc]}\chi^\beta \tag{22}$$

where $V_\alpha \equiv D_\alpha\varphi$ is the gravitello superfield, $T_{abc} = T_{ab}{}^d\eta_{dc}$ belongs to the 120 irrep. and $T_{bc}{}^\beta$ is the gravitino field strength. Moreover, the eqs. (13)–(17) imply the following relations

$$D_\alpha V_\beta = -\Gamma^a_{\alpha\beta}D_a\varphi + V_\alpha V_\beta + \frac{1}{12}(\Gamma_{abc})_{\alpha\beta}\left(T^{abc} + \frac{\gamma}{2}\chi^{abc}\right) \tag{23a}$$

$$D_\alpha T_{abc} = (\Gamma_{[a})_{\alpha\beta}\left(-6T_{bc]}{}^\beta - 12\gamma F_{bc]}\chi^\beta\right) \tag{23b}$$

$$D_\alpha\chi^\beta = \frac{1}{4}(\Gamma_{ab})_\alpha{}^\beta F^{ab} + T_{\gamma\alpha}{}^\beta\chi^\gamma \tag{23c}$$

$$D_\alpha F_{ab} = 4(\Gamma_{[a})_{\alpha\beta}D_{b]}\chi^\beta + (\Gamma^{cd}{}_{[a})_{\alpha\beta}T_{b]cd}\chi^\beta \tag{23d}$$

$$D_{[a}T_{bcd]} + \frac{3}{2}T^f{}_{[ab}T_{cd]f} = \frac{3\gamma}{2}F_{[ab}F_{cd]} \tag{24}$$

$$D_cT^c{}_{ab} = 2D_c\varphi T^c{}_{ab} + 2T_{ab}{}^\alpha V_\alpha + \gamma\left(4F_{ab}\chi^\alpha V_\alpha - 4\chi_\alpha(\Gamma_{[a})^{\alpha\beta}D_{b]}\chi_\beta + T_{[a}{}^{cd}\chi_{b]cd}\right), \tag{25}$$

together with the field equations:

$$(\Gamma^a)^{\alpha\beta}D_aV_\beta = 2(\Gamma^a)^{\alpha\beta}D_a\varphi V_\beta - \frac{1}{12}(\Gamma^{abc})^{\alpha\beta}T_{abc}V_\beta + \gamma\left(2\chi^\alpha\chi^\beta V_\beta - \frac{1}{2}(\Gamma^{ab})_\beta{}^\alpha F_{ab}\chi^\beta\right) \tag{26}$$

$$D^a D_a\varphi = 2D^a\varphi D_a\varphi - \frac{1}{12}T^{abc}T_{abc}$$

$$+\gamma\left(\frac{1}{12}T^{abc}\chi_{abc} - \frac{1}{96}\chi^{abc}V_{abc} - \frac{1}{4}F^{ab}F_{ab} + \frac{1}{4}(\Gamma^{ab})_\alpha{}^\beta V_\beta\chi^\alpha F_{ab}\right) \tag{27}$$

$$(\Gamma^b)_{\alpha\beta}T_{ba}{}^\beta = D_aV_\alpha + \frac{1}{4}(\Gamma^{bc})_\alpha{}^\beta V_\beta T_{abc} - \gamma(\Gamma^b)_{\alpha\beta}F_{ba}\chi^\beta \tag{28}$$

$$R_{(ab)} = 2D_{(a}D_{b)}\varphi + \gamma\left(2\chi^\alpha(\Gamma_{(a})_{\alpha\beta}D_{b)}\chi^\beta - F_a{}^cF_{bc} + \frac{1}{2}\chi_{(a}{}^{cd}T_{b)cd}\right) \tag{29}$$

$$(\Gamma^a)_{\alpha\beta}D_a\chi^\beta = \left((\Gamma^b)_{\alpha\beta}D_b\varphi + V_\alpha V_\beta - \frac{1}{6}(\Gamma^{abc})_{\alpha\beta}T_{abc}\right)\chi^\beta - \frac{1}{4}(\Gamma^{ab})_\alpha{}^\beta F_{ab}V_\beta \tag{30}$$

$$D^bF_{ba} = 2D^b\varphi F_{ba} + 2V_\beta D_a\chi^\beta + T_{abc}F^{bc} - \frac{1}{2}T_{abc}(\Gamma^{bc})_\beta{}^\gamma V_\gamma\chi^\beta. \tag{31}$$

Here we defined

$$V^{abc} = (\Gamma^{abc})^{\alpha\beta}V_\alpha V_\beta.$$

Eqs. (26) – (31) are the field equations for the gravitello, the dilaton, the gravitino, the graviton, the gaugino and the gauge boson respectively.

According to ref. [3], using the relations above, one can define a 3–superform H_3 with components

$$\begin{aligned} H_{abc} &= T_{abc} \\ H_{a\alpha\beta} &= 2(\Gamma_a)_{\alpha\beta}, \end{aligned} \tag{32}$$

and all others vanishing, such that (see for instance (24))

$$dH_3 = \gamma F_2F_2, \tag{33}$$

and a 7–superform H_7 with components

$$H_{a_1\cdots a_7} = \frac{1}{3!}e^{-2\varphi}\varepsilon_{a_1\cdots a_7}{}^{b_1b_2b_3}(H_{b_1b_2b_3} - V_{b_1b_2b_3} - \gamma\chi_{b_1b_2b_3}) \tag{34a}$$

$$H_{\alpha a_1\cdots a_6} = -2e^{-2\varphi}(\Gamma_{a_1\cdots a_6})_\alpha{}^\beta V_\beta \tag{34b}$$

$$H_{\alpha\beta a_1\cdots a_5} = -2e^{-2\varphi}(\Gamma_{a_1\cdots a_5})_{\alpha\beta} \tag{34c}$$

and all others vanishing, such that

$$dH_7 = 0. \tag{35}$$

It follows from (33) and (35) that one can write locally

$$H_3 = dB_2 + \gamma A_1 F_2 \tag{36}$$

and

$$H_7 = dB_6. \tag{37}$$

Of course B_2 and B_6 are not independent superforms since their curvatures are related through eq. (34a) and can be considered as dual one to the other. If one works with B_2 eq. (33) is the B.I. and eq. (35) provides the field equation of B_2 and, viceversa, working with B_6 eq. (35) is the B.I. and eq. (33) contains the field equation of B_6.

Going to the gauge sector, let us notice at first that the gauge boson field equation (31), using the gluino, gravitello and gravitino equations of motion, can be rewritten as

$$e^{2\varphi} D^b \left(e^{-2\varphi} \tilde{F}_{ba} \right) - \frac{1}{2} T_{abc} \tilde{F}^{bc} - (\Gamma_a{}^{bc})_{\alpha\beta} T_{bc}{}^{\beta} \chi^{\alpha} = \frac{1}{2} (H_{abc} - V_{abc} - \gamma \chi_{abc}) F^{bc} \tag{38}$$

where

$$\tilde{F}^{ab} \equiv F^{ab} + 2(\Gamma^{ab})_{\alpha}{}^{\beta} V_{\beta} \chi^{\alpha}.$$

Now let us define the 8–superform F_8 through

$$F_{a_1 \cdots a_8} = \frac{1}{2} e^{-2\varphi} \varepsilon_{a_1 \cdots a_8}{}^{b_1 b_2} \tilde{F}_{b_1 b_2} \tag{39a}$$

$$F_{\alpha a_1 \cdots a_7} = -2 e^{-2\varphi} (\Gamma_{a_1 \cdots a_7})_{\alpha\beta} \chi^{\beta}, \tag{39b}$$

and all other components vanishing. A lengthy but straightforward calculation shows that F_8 satisfies the identity

$$dF_8 = F_2 H_7, \tag{40}$$

so that locally there exists a 7–superform A_7 such that

$$F_8 = dA_7 + F_2 B_6. \tag{41}$$

The simplest way to prove (40) is to define $Y_9 \equiv dF_8 - F_2 H_7$. Y_9 vanishes trivially in the sectors (5,4), (4,5),$\cdots$,(0,9). A relatively easy calculation shows that it vanishes also in the sectors (6,3) and (7,2). Then, looking at the identity $dY_9 = 0$ in the sectors (7,3) and (8,2), one can see immediately that Y_9 vanishes also in the sectors (8,1) and (9,0). However, it is instructive to verify directly that (the dual of) eq. (40) in the sector (9,0) is precisely eq. (38).

As for the dilaton sector, the use of the equation of motion of the gravitino allows to rewrite the dilaton field equation (27) in the form

$$\begin{aligned} e^{2\varphi} D_a \left(e^{-2\varphi} D^a \varphi\right) - \frac{1}{2} (\Gamma^{ab})_\alpha{}^\beta T_{ab}{}^\alpha V_\beta = & -\frac{1}{12} H_{abc} \left(H^{abc} - V^{abc} - \gamma \chi^{abc}\right) \\ & - \frac{\gamma}{4} F_{ab} \tilde{F}^{ab}. \end{aligned} \tag{42}$$

Then we can define the 9–superform V_9 with components

$$V_{a_1 \cdots a_9} = 2 e^{-2\varphi} \varepsilon_{a_1 \cdots a_9}{}^b D_b \varphi \tag{43a}$$

$$V_{\alpha a_1 \cdots a_8} = -2 e^{-2\varphi} (\Gamma_{a_1 \cdots a_8})_\alpha{}^\beta V_\beta \tag{43b}$$

and all others vanishing, and verify, as before, that V_9 satisfies the identity

$$dV_9 = H_3 H_7 + \gamma F_2 F_8. \tag{44}$$

The (dual of) eq. (44) in the sector (10,0) is just the dilaton field equation (42). It follows from (44) that locally there exists an 8–superform φ_8 such that

$$V_9 = d\varphi_8 - B_2 H_7 + \gamma A_1 F_8, \tag{45}$$

or, alternatively, $\hat{\varphi}_8$ such that

$$V_9 = d\hat{\varphi}_8 + H_3 B_6 + \gamma F_2 A_7. \tag{46}$$

In models with $2n$ Super–Maxwell multiplets, some of the field equations and supersymmetry transformations become slightly more complicated (see, for instance [10] where a set of constraints similar to ours has been used). However, it is straightforward to extend our results to this case to get again eqs. (7) – (11) where the $F_8^{(r)}$ are still given by eq. (39) for each r ($r = 1, \cdots, 2n$).

As B_2 and B_6 in the B sector, also A_1 and A_7 in the gauge sector are not independent superforms and must be considered as dual one to each other. The same happens for φ and φ_8 in the dilaton sector. Then one can foresee different formulations of $N = 1, D = 10$ SUGRA–MAX models (with gauge group $U(1)^{2n}$) where B_2 or B_6, $A_1^{(r)}$ or $A_7^{(r)}$ for each r, φ or $(\varphi_8, \hat{\varphi}_8)$ are considered as the fundamentals fields. If one chooses $A_1^{(r)}$ ($A_7^{(r)}$) eq. (2) (eq. (7)) is the B.I. and eq. (7) (eq. (2)) provides the relevant equation of motion. Similarly for φ $(\varphi_8, \hat{\varphi}_8)$ eq. (12) (eq. (9)) is the B.I. and eq. (9) (eq. (12)) provides the relevant equation of motion.

However, notice that not all combinations of B_2 or B_6, A_1 or A_7, φ or φ_8 are allowed. Indeed, taking a look on eq. (8) one sees that A_7 is compatible with B_6 but not with B_2. As for $(\varphi_8, \hat{\varphi}_8)$, it is convenient to rescale $B_6, A_7, (\varphi_8, \hat{\varphi}_8)$ as well as H_7, F_8, V_9 by the factor $e^{2\varphi}$ in order to get rid of any dependence on φ in eqs. (45),(46) and (41). Then one can see that φ_8 is compatible only with B_2, A_1 while $\hat{\varphi}_8$ is compatible only with B_6, A_7.

Among the allowed formulations that in terms of (φ, A_1, B_2) is the standard formulation and that in terms of (φ, A_1, B_6) is the "old" dual one. The new formulation advocated in ref. [1] corresponds to the one formulated in terms of $(\varphi, B_6, A_1^{(r)}, A_7^{(r+n)}, r = 1, \cdots, n)$. The previous remark about the compatibility of A_7 with B_6 but not with B_2 is in agreement with the result of [1], namely that the ten-dimensional supergravity version which leads to the manifestly $SL(2, R)_S$ invariant action, after toroidal compactification to four dimensions, is the one which involves B_6.

We must point out a complication that arises when working with A_7 instead of A_1: when in eq. (41) F_2 is removed in favour of F_8, eq. (41) cannot be inverted in a closed form to get F_8 in terms of A_7. The best one can do is to express F_8 in terms of A_7 as an iterative series. A similar complication arises in the dilaton sector, eqs. (45),(46). Nevertheless this situation is not new; it was also met for instance in the equation for H_{abc} of the anomaly free models with Lorentz Chern–Simons coupling, see [11].

Another feature of eq. (41) has to be pointed out: in order to get an F_8 invariant under the gauge transformation of B_6,

$$\delta B_6 = d\Lambda_5, \tag{47}$$

A_7 too has to transform as

$$\delta A_7 = -F_2 \Lambda_5. \tag{48}$$

This is analogous to the fact that B_2 in the usual formulation, transforms under gauge (and Lorentz) transformations as a consequence of the gauge (and Lorentz) Chern–Simons forms in the definition of H_3. However, here (41) is invariant under (47) and (48) only if eq. (2) is satisfied, which now has to be interpreted as the equation of motion of A_7, so that invariance of eq. (41) holds only on–shell.

Let us comment finally on what our results become in two more general situations. If one considers a non abelian gauge group eq. (40) does no longer hold since in this case the gluinos are minimally coupled to the gluons and (40) becomes now $DF_8 = F_2 H_7 + j_9$ where j_9 is a current nine-superform which is Lie-algebra-valued together with F_2 and F_8. In this case it is not possible to replace A_1 by A_7, nor to replace φ by $(\varphi_8, \hat{\varphi}_8)$. Nevertheless eq. (44) holds again upon tracing the F-terms on its right hand side. The r.h.s. of (44) is in this case a closed form, thanks to the identity $tr(j_9 F_2) = 0$, but not an exact one as in the abelian case.

On the contrary, if one keeps the gauge group abelian but introduces the Lorentz-Chern-Simons three form in the definition of H_3 (which amounts to a modification of J_{cde} in (18)) it can be seen that (40) holds again upon substituting the term $\gamma\chi_{b_1 b_2 b_3}$ in (34a) – and in due places – with $-12 J_{b_1 b_2 b_3}$. In fact, the sectors $(6,3)$ and $(7,2)$ of Y_9, defined after eq. (41), are independent on J_{abc} and its superspace derivatives, and again $dY_9 = 0$ due to $dH_7 = 0$. On the other hand now (44) does no longer hold. Therefore in this case it is still possible to describe the gauge degrees of freedom in terms of A_7, while the dilaton has to be described by a scalar.

It may also be that our results shed some new light on the string/fivebrane or string/string duality relations conjectured recently and in the past (see ref. [12] and references therein).

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